NYCIML
SENIOR B DIVISION
SPRING 1992

CONTEST 1

PART 1: 10 MINUTES

S92B1 150 coins are arranged in piles so that each pile has one more than the previous pile. If the smallest pile has three coins, how many piles are there?

S92B2 If $4^x = \sqrt{2^{3y}}$ compute the numerical value of x/y.

PART 2: 10 MINUTES

S92B3 If everyone in a room shakes hands with everyone else, and 153 handshakes take place, compute the number of people in the room.

S92B4 Find the number of positive integral solutions for $2x + 3y = 1009$.

PART 3: 10 MINUTES

S92B5 A regular dodecagon (12-sided polygon) is inscribed in a circle of radius 5. Compute the area of the dodecagon.

S92B6 Find the smallest positive integer x for which $3150x = N^3$, where N is an integer.

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CONTEST 2

PART 1: 10 MINUTES

S92B7 Express $\frac{1-\cos 2x}{\sin 2x}$ as a single trigonometric function.

S92B8 Compute the number of ways four men and four women can be seated at a circular table if the men and women sit alternately?

PART 2: 10 MINUTES

S92B9 Compute the sum of the infinite series .06+.006+.0006+.00006+…

S92B10 Jane can row five miles per hour in still water. Rowing 40 miles with the current, it takes her only one half the time that it takes to row the same distance against the current. Compute the rate of the current.

PART 3: 10 MINUTES

S92B11 The decimal numeral 1000 is written as x in the base 3. Find x.

S92B12 The perimeter of an isosceles triangle is 20 and the altitude to the base has length 5. Compute the area of the triangle.

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CONTEST 3

PART 1: 10 MINUTES

S92B13 If the sum of two numbers is twenty and their product is thirty, compute the sum of the reciprocals.

S92B14 The lengths of the sides of a triangle are 3, 7 and 8. If an altitude is dropped to the longest side, compute the length of the longer segment formed on this side.

PART 2: 10 MINUTES

S92B15 Several women are walking their dogs. If the total number of legs is 22 more than twice the number of heads, compute the number of dogs.

S92B16 If $8^{x+3} = 8^x + 1022$, compute the value of x.

PART 3: 10 MINUTES

S92B17 Find all ordered pairs (x, y) of positive integers that solve the equation $26x + 25y = 1000$.

S92B18 Rationalize the denominator: $\dfrac{\sqrt{2}}{\sqrt{2}+\sqrt{3}-\sqrt{5}}$.

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CONTEST 4

PART 1: 10 MINUTES

S92B19 Find the remainder when 10^{61} is divided by 7.

S92B20 An old printer can print a newspaper in ten hours, while a new one can print the same newspaper in six hours. If four old printers and three new printers work simultaneously, compute how long it would take to print a newspaper.

PART 2: 10 MINUTES

S92B21 Compute the numerical value of x if $x = \log_{3x} 36$.

S92B22 Compute the value of the infinite series: $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \frac{1}{32} + \ldots$.

PART 3: 10 MINUTES

S92B23 Find the ordered pair (x, y), which will satisfy the equation $x^2 + 10x + y^2 - 12y + 61 = 0$.

S92B24 A point P is chosen in the interior of rectangle ABCD so that PA=3, PB=4, and PC=5. Find PD.

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CONTEST 5

PART 1: 10 MINUTES

S92B25 The average of thirty students' marks on a mathematics test is 80. Compute the average of the remaining students if David's 97, Jeffrey's 75, and Anne's 95 are not counted.

S92B26 If $73x$ is twice $36x$ (where these are two numbers in base x), find x.

PART 2: 10 MINUTES

S92B27 Find all real values of x which satisfy the equation $\log_{10}(x^2+5x-50)=2$.

S92B28 If $\sin\left(\frac{\pi}{2}\right)=\frac{1}{4}$, compute the value of $\cos 2x$.

PART 3: 10 MINUTES

S92B29 Two externally tangent circles have radii with lengths 4 and 6. Compute the length of a common external tangent segment.

S92B30 Compute the number of positive integer solutions (a, b) of the equation: $a^2-b^2=105$.

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CONTEST 1 - SOLUTIONS

S92B1 Using the formula for the sum of an arithmetic sequence,
$S=\frac{n}{2}\left(2a+(n-1)d\right)$, $150=\frac{n}{2}\left(6+(n-1)\right)$, $300=5n+n^2$,
$(n+20)(n-15)=0$, so $n=15$.

S92B2 Rewriting the problem using powers of 2 we get: $2^{2x}=2^{3y/2}$, which implies
$4x=3y$ or $\frac{x}{y}=\frac{3}{4}$.

S92B3 $153=\frac{n(n-1)}{2}\rightarrow n^2-n-306=0$ or $n=18$.

S92B4 Solving for x, $x=\frac{1009-3y}{2}$. Therefore, $3y\le 1009$ and 336 integers will fit the equation; however, y must be odd in order to make x an integer. Thus, there are half as many solutions or 168 solutions.

S92B5 Using 12 radii, there are 12 triangles, each of which has an area of
$\frac{1}{2}(5)(5)\sin 30^\circ=\frac{25}{4}$. Multiplying by 12, we get an area of 75 square units.

S92B6 Since $3150=2^1 3^2 5^2 7$, x must equal $2^2 3^1 5^1 7^2=2940$, so that
$3150x=2^3 3^3 5^3 7^3=210^3$.

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CONTEST 2 - SOLUTIONS

S92B7 $\frac{1-\cos 2x}{\sin 2x} = \frac{1-(1-2\sin^2 x)}{2\sin x \cos x} = \frac{2\sin^2 x}{2\sin x \cos x} = \tan x$

S92B8 Placing the men first in alternate seats, there are 3!=6 ways that these men can sit. Once the men are seated, there are 4! ways of placing the women. Thus, there are (6)(24) or 144 ways of seating.

S92B9 This is an infinite geometric progression with a common ratio of 0.1 so that $S = \frac{0.06}{1-0.1} = \frac{6}{90} = \frac{1}{15}$.

S92B10 Using the fact that the time it takes is equal to the quotient of the total distance gone and the rate of the boat and current (assuming a constant rate) we get: $\frac{40}{5-c} = 2\cdot\left(\frac{40}{5+c}\right)$ so that 400-80c = 200 +40c or $c = \frac{5}{3}$.

S92B11 Changing the base 10 of a number to base 3 can be done by taking the remainders when divided by 3 in reverse order.

Number	**Remainder**
1000	
333	1
111	0
37	0
12	1
4	0
1	1
0	1

Thus, the number in base 3 is 1101001.

S92B12 $2x+2y=20 \to x+y=10$

$x^2+5^2=y^2$

$x^2+5^2=(10-x)^2$

$x^2+25=100-20x+x^2$

$20x=75 \to x=3.75$

$A=5x \to A=18.75$

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CONTEST 3 - SOLUTIONS

S92B13 Let the numbers be represented by x and y. Thus, $\frac{1}{x}+\frac{1}{y}=\frac{x+y}{xy}=\frac{20}{30}=\frac{2}{3}$.

S92B14

$$x^2+y^2=49$$
$$(8-x)^2+y^2=9$$
$$49-x^2=9-(8-x)^2$$
$$49-x^2=9-64+16x-x^2$$
$$16x=104$$
$$x=6\frac{1}{2}$$

S92B15 If d = the number of dogs and w = the number of women walking them.
4d + 2w = 22 + 2(d + w)
4d = 22 + 2d $\rightarrow$ d = 11

S92B16 $8^{x+2}-8^x=1022 \rightarrow 8^x(8^3-1)=1022 \rightarrow 8^x(511)=1022 \rightarrow 8^x=2$
$x=\frac{1}{3}$.

S92B17 $y=\frac{1000-26x}{25}=40-\frac{26x}{25}$
X must therefore be divisible by 25, any x value 50 or more would make y negative. Thus, the only solution is (25, 14).

S92B18 $\frac{\sqrt{2}}{\sqrt{2}+\sqrt{3}-\sqrt{5}}\cdot\frac{\sqrt{2}+\sqrt{3}+\sqrt{5}}{\sqrt{2}+\sqrt{3}+\sqrt{5}}=\frac{2+\sqrt{6}+\sqrt{10}}{2\sqrt{6}}$

Multiplying by $\frac{\sqrt{6}}{\sqrt{6}}$ gives: $\frac{3+\sqrt{6}+\sqrt{15}}{6}$.

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CONTEST 4 - SOLUTIONS

S92B19

$10^2 = 100 \equiv 2 \pmod 7$

$10^6 = (10^2)^3 \equiv 2^3 \pmod 7 \equiv 1 \pmod 7$

$10^{60} = (10^6)^{10} \equiv 1^{10} \equiv 1 \pmod 7$

$10^{60}(10) \equiv 10 \pmod 7 \equiv 3 \pmod 7$ giving 3.

S92B20

Let x = the amount of time it takes together.

$\frac{4x}{10} + \frac{3x}{6} = 1$ so that $x = \frac{10}{9} = 1\frac{1}{9}$.

S92B21

$(3x)^x = 36$ so that by inspection, x = 2.

S92B22

Separating into two series using alternate terms we get:

$1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{32} + ... = \frac{1}{1 - \frac{1}{4}} = \frac{4}{3}$

$\frac{1}{2} + \frac{1}{8} + \frac{1}{32} + ... = \frac{\frac{1}{2}}{1 - \frac{1}{4}} = \frac{2}{3}$

Thus, we get $\frac{4}{3} - \frac{2}{3} = \frac{2}{3}$.

S92B23

Completing the squares yields:

$(x+5)^2 + (y-6) = -61 + 25 + 36 = 0$

Thus, the answer is (-5, 6).

S92B24

Through P, draw a line parallel to the bases.

$$\begin{array}{l} 5^2 - b^2 = x^2 - a^2 \\ \underline{4^2 - b^2 = 3^2 - a^2} \\ 9 = x^2 - 9 \\ x^2 = 18 \\ x = 3\sqrt{2} \end{array}$$

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CONTEST 5 - SOLUTIONS

S92B25 The sum of the 30 marks is 2400. The sum of the remaining 27 marks is 2133 so that their average is 79.

S92B26 7x + 3 = 2(3x + 6) $\rightarrow$ x = 9

S92B27 $10^2 = x^2 + 5x - 50 \rightarrow x^2 + 5x - 150 = 0$, giving x = $\{10, -15\}$.

S92B28 Using the double angle formula, $\cos x = 1 - 2\sin^2 \frac{x}{2} = 1 - 2(1/4)^2 = 7/8$.
$\cos 2x = 2\cos^2 x - 1 = 2\left(\frac{7}{8}\right)^2 - 1 = \frac{17}{32}$.

S92B29 The line of centers $(\overline{OP})$ is 10 units long. Drawing $\overline{OT}$ parallel to $\overline{RS}$ and using the Pythagorean Theorem, we get $OT = \sqrt{96}$. Since OTSR is a rectangle, $RS = \sqrt{96}$.

S92B30 Factoring, (a + b)(a – b)=105. The only pairs of factors of 105 are 105, 1; 35, 3; 21, 5 and 15, 7. Each pair of equations (a + b = 105 and a – b = 1) will produce a valid solution: (53, 52), (19, 16), (13, 8), and (11, 4) or 4 solutions.