**NYCIML**
**JUNIOR DIVISION**
**FALL 1992**

## CONTEST 1

## PART 1: 10 MINUTES

F92J1 At a certain high school reunion, everyone shook hands with everyone else exactly once. If there were 3160 handshakes, how many people attended this reunion?

F92J2 Compute all values of $\frac{1}{x}$ such that $2-\frac{7}{x}+\frac{6}{x^2}=0$.

## PART 2: 10 MINUTES

F92J3 Compute the total number of divisors of $3^2 \cdot 5^3 = 1125$.

F92J4 If A, B, and C are vertices of the cube show, compute the degree measure of $\angle CBA$.

## PART 3: 10 MINUTES

F92J5 In a certain official class, there are 32 students who speak Mandarin, Cantonese or Toishanese (3 Chinese dialects). No one speaks Cantonese only or Toishanese only. The number who speak Mandarin only is equal to the number speaking Cantonese and Mandarin only. The number speaking Cantonese and Toishanese only is four times the number speaking all three. (At least one person speaks all three.) There are four who speak Toishanese and Mandarin only. If the number speaking Cantonese and Toishanese only is less than the number speaking Mandarin and Cantonese only, how many speak Mandarin only?

F92J6 The numbers 3, 4, 5, 6, 7, 8, and 9 are placed in a hat. If four numbers are drawn one after another, WITHOUT replacement, compute the probability that the product of the number chosen is odd.

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**CONTEST 2**

**PART 1: 10 MINUTES**

F92J7 Compute all values of 1/x such that $\frac{4}{x} - \frac{5}{x^3} + \frac{1}{x^5} = 0$.

F92J8 Of all students taking a test, those that passes had an average grade of 74, while those that failed had an average grade of 58. If the average of all students was 70, compute the probability that a randomly chosen student in this class passed the test.

**PART 2: 10 MINUTES**

F92J9 Compute the total number of positive divisors of 65219.

F92J10 Hexagon ABCDEF is formed by connecting the midpoints of adjacent sides of the unit cube shown. (Each side has a length of one.) Compute the area of hexagon ABCDEF.

**PART 3: 10 MINUTES**

F92J11 A commuter train runs along a straight line path. It starts at town A and stops at towns B, C, D, and E before arriving at city F where it reverses itself and continues to go towards A, back and forth all day. Of the distances between adjacent stops are equal, compute the probability that the train is either about to arrive or leave from F.

F92J12 What is the remainder when $2^{150}$ is divided by 9?

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**CONTEST 3**

**PART 1: 10 MINUTES**

F92J13 Compute the total number of divisors of $3^4 \cdot 2 \cdot 7 \cdot 10^2$.

F92J14 Compute all real values of 1/x such that $\frac{64}{x^2} - \frac{4}{x^6} - \frac{16}{x^4} + \frac{1}{x^8} = 0$.

**PART 2: 10 MINUTES**

F92J15 Find the volume of the pyramid PEFGH where P is the intersection of the diagonals AG and FD of the unit cube shown.

F92J16 A commuter train runs along a straight line path. It starts at town A and stops at towns B, C, D and E before arriving at city F where it reverses itself and continues to go towards A, back and forth all day. Distances between adjacent stops are equal. There is only ONE person waiting for a train on the whole commuter line! Compute the probability that when the train arrives, it is headed in the wrong direction for this person.

**PART 3: 10 MINUTES**

F92J17 Nick runs twice as fast as he walks. One day, he walks for twice the time he runs. It takes twenty minutes to get to school. A week later, keeping the same pace, he runs for twice the time he walks. How many minutes does it take Nick to get to school the second time?

F92J18 A professor can assign only A, B, C, or F as grades in a special course. Of she decides to give an equal amount of each grade, in how many ways can she assign grades in a class of 12 students?

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**CONTEST 1 – SOLUTIONS**

F92J1 For n people, there are ${}_nC_2$ ways of choosing 2 people for a handshake. This gives the equation $\frac{n(n-1)}{2}=3160$, which gives $n^2-n-6320=0$, $(n-80)(n+79)=0$. Thus there are 80 people.

F92J2 Factoring gives $\left(2-\frac{3}{x}\right)\cdot\left(1-\frac{2}{x}\right)=0$. Solving for $1/x$ gives 2/3 and ½.

F92J3 To find the number of divisors of a number, first write the prime factorization of the number. For N = $x^a y^b$, where x and y are both prime, the number of positive divisors of N is (a + 1) (b + 1). In this problem we get 3(4) = 12 divisors.

F92J4 The six faces of a cube are congruent and so are the diagonals of the six faces. Connecting AC gives an equilateral triangle, each of whose angles contain sixty degrees.

F92J5 The Venn diagram on the right uses the given information using x as the number who speak Mandarin only and y as the number who speak all three. The fact that there are 32 students gives the equation x + x + 4 + y + + 4y = 32 which reduces to 2x + 5y = 28. The only values of x yielding positive integers for y are x = 4 and x = 9. If x = 4, y = 4. The last statement of the problem indicates that 4y < x, which does not hold if x = 4. If x = 9, y = 2 , for which 4y < x. Thus, there are 9 people who speak Mandarin only.

F92J6 The probability can be found by multiplying:
$\frac{4}{7}\cdot\frac{3}{6}\cdot\frac{2}{5}\cdot\frac{1}{4}$ which yields $\frac{1}{35}$.

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**CONTEST 2 – SOLUTIONS**

F92J7 Factoring gives $\frac{1}{x}\left(4-\frac{1}{x^2}\right)\left(1-\frac{1}{x^2}\right)=0$. Note that the first factor can never equal zero. Setting each of the other factors equal to zero yields the results $\frac{1}{x}=\pm1,\pm2$.

F92J8 Let x be the number of students who passed and y be the number of students who failed. The sum of all the marks in the class is 74x + 58y. Thus, the average mark is $\frac{74x+58y}{x+y}=70$. This simplifies to $\frac{16x}{x+y}+\frac{58x+58y}{x+y}=70$ or $\frac{16x}{x+y}+58=70 \to 16x=12(x+y) \to 4x=12y$ or $x=3y$. Thus, three times as many students passed as failed meaning the probability of passing is <u>¾ = .75.</u>

F92J9 65219 is obviously divisible by 11 since 6-5+2-1+9 is divisible by 11. 65219 = 11(5929) = 11(11)(539) = 11(11)(11)(49). Thus, the prime factorization of 65219 is $11^3\cdot7^2$. Using the theorem on the number of positive divisors of a number, we get 4(3) = <u>12 divisors.</u>

F92J10 Hexagon ABCDEF is formed by connecting midpoints of adjacent sides in the unit cube. Thus, each side of ABCDEF has length $\frac{\sqrt{2}}{2}$. A hexagon can be broken up into six congruent triangles by connecting opposite vertices. Each triangle is equilateral having sides with length $\frac{\sqrt{2}}{2}$. Using the formula for the area of an equilateral triangle, each triangle has area $\frac{s^2\sqrt{3}}{4}$. Thus, the area of the hexagon is six times this result, which is $\frac{1}{2}\cdot\frac{\sqrt{3}}{4}\cdot6=\frac{3\sqrt{3}}{4}$.

F92J11 When the train is in between stations, we can describe its relative position using ordered pairs. Thus, for example, (A, B) means the train is in between A and B and headed for B, while (B, A) means the train is headed for A from B. The possible positions are (A, B), (B, C), (C, D), (D, E), (E, F), (F, E), (E, D), (D, C), (C, B), and (B, A). Out of the ten possible positions, two of them involve arriving at or leaving from F. Thus the probability is <u>2/10.</u>

F92J12 $2^3 \equiv -1 \pmod 9$. Raising both sides of this equation to the 50th power yields $2^{150} \equiv +1 \pmod 9$. Thus, the remainder on division by 9 is <u>+1.</u>

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**CONTEST 3 – SOLUTIONS**

F92J13 Note that the given number was not factored into primes. The prime factorization is $3^4 \cdot 2^3 \cdot 5^2 \cdot 7$. Thus, the number of positive divisors is $5 \cdot 4 \cdot 3 \cdot 2$ = <u>120.</u>

F92J14 Factoring gives $\frac{1}{x^2}\left(16-\frac{1}{x^4}\right)\left(4-\frac{1}{x^2}\right)$, which yields $\frac{1}{x}=\pm 2$.

F92J15 The diagonals of a cube are congruent and bisect each other. A diagonal of a face has length $\sqrt{2}$, while a major diagonal of the cube has length $\sqrt{3}$. Thus, PE = $\frac{\sqrt{3}}{2}$. Using the Pythagorean Theorem to find the altitude PX, we get $PX^2+\frac{1}{2}=\frac{3}{4}$ so that $PX=\frac{1}{2}$. Using the formula for the volume of a pyramid, $V=\frac{Bh}{3}$ m where h is the altitude and B is the base area, we get $V=\frac{1}{3}\cdot 1\cdot\frac{1}{2}=\frac{1}{6}$.

F92J16 Several approaches are possible. One such approach: If the person is at A or F, the arriving train should be considered as going in the correct direction since the commuter cannot be <u>*headed*</u> in the wrong direction. (The next stop is B or E.) At B, C, D and E, the train will be headed wrong half of the time. Simplify by considering ten people per station. On the average, half at B, C, D, and E (or 20 people) will head in the wrong direction. No one at A or F is headed wrong. Thus, twenty out of 60 times people are randomly given a station, a train heads the wrong way. Thus the probability is 1/3.

F92J17 Remember, the total distance traveled = AVERAGE rate times the total amount of time. Letting x be Nick's average walking rate, 2x his average running rate, and t his time of running the first day and 2t the time walking, we get 2tx for both the distance he walked and the distance he ran. The total time going to school is 3t = 20 minutes, giving t = 20/3 and a TOTAL distance to school 80x/3. In the second situation let y be the walking time and 2y the running time. This gives (2x)(2y) + xy = 80x/3.

Since x is not zero, we divide and get 5y = 80/3. The total time is 3y = 80/5 = <u>16 minutes.</u>

F92J18 The number of ways to assign grades is

$${}_{12}C_3 \cdot {}_9C_3 \cdot {}_6C_3 \cdot {}_3C_3 = \frac{12\cdot 11\cdot 10}{3\cdot 2\cdot 1}\cdot\frac{9\cdot 8\cdot 7}{3\cdot 2\cdot 1}\cdot\frac{6\cdot 5\cdot 4}{3\cdot 2\cdot 1}\cdot 1 = \underline{369{,}600.}$$