NYCIML
SENIOR B DIVISION
FALL 1992

CONTEST 1

PART 1: 10 MINUTES

F92B1 Express $\dfrac{\log 5}{\log(1/5)}$ in simplest form.

F92B2 If the first term of an arithmetic progression is r and the common difference is 2r, express in terms of r and n, the sum of the first n terms.

PART 2: 10 MINUTES

F92B3 Find all roots of the equation $\sqrt{32-x^2}+x=0$.

F92B4 The 25 teams in the little league were divided into two divisions. Each team played every team in its division exactly once, and did not play any team in the other division. If the number of games played in division A was 36 more than the number of games played in division B, how many teams were in Division A?

PART 3: 10 MINUTES

F92B5 Triangle ABC is inscribed in circle O. Arc AB=130° and arc BC=80°. Point E is chosen on minor arc AC so that radius OE is perpendicular to AC. Find the measure of angle OBE.

F92B6 Find all ordered pairs of integers (x, y) which satisfy $2^{2x}-3^{2y}=55$.

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CONTEST 2

PART 1: 10 MINUTES

F92B7 The average of Jeff's nine test grades is 92. If his highest grade, 96, and his lowest grade are <u>not</u> counted, the average of the other seven grades is 94. What was his lowest grade?

F92B8 How many ordered pairs of positive integers (x, y) are there which satisfy the equation $3x + 5y = 500$?

PART 2: 10 MINUTES

F92B9 Find all x for which the reciprocal of $x + 1$ is $x-1$.

F92B10 What is the total number of positive factors of 1728?

PART 3: 10 MINUTES

F92B11 Find the two roots of

$$\left(x-\frac{1}{4}\right)\left(x-\frac{1}{4}\right)+\left(x-\frac{1}{4}\right)\left(x-\frac{1}{8}\right)=0 .$$

F92B12 The number 12! Is multiplied out and then changed to base 12. How many zeros does the number end with?

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CONTEST 3

PART 1: 10 MINUTES

F92B13 Find the distance between the two points of intersection of the graphs $x^2 + y = 7$ and $x + y = 7$.

F92B14 A regular octagon is formed by cutting equal isosceles right triangles from the corners of a square with side 4. Find the length of a leg of one of these triangles.

PART 2: 10 MINUTES

F92B15 In terms of p and q, find the sum of the reciprocals of the roots of the equation $x^2 + px + q = 0$.

F92B16 If $[x]$ represents the greatest integer function, find the smallest value of x for which $[x]+[3x]+[5x]=21$.

PART 3: 10 MINUTES

F92B17 If $\sin x = \frac{1}{4}$, find all possible values of $\sin 2x$.

F92B18 Two chords are perpendicular in a circle. One has segments of 4 and 3 and the other has segments 2 and 6. Find the radius of the circle.

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CONTEST 4

PART 1: 10 MINUTES

F92B19 If $7^{3x} = 216$. Find the value of 7^{-x}.

F92B20 A newspaper office has two presses. An old one can print the newspaper in 6hours, and a new one can print the newspaper in 5 hours. After they have both been working for 2 hours, the old one breaks down. How long would it take the new one to finish the job?

PART 2: 10 MINUTES

F92B21 A regular octagon is inscribed in a circle with radius 10. Find the area of the octagon.

F92B22 The sum of an infinite geometric progression is 4 and the sum of the first 2 terms is 3. Find all possible values of the first term.

PART 3: 10 MINUTES

F92B23 The price of an article was increased by P%. The new price was decreased by P%. If the final price was one dollar, express, in terms of P, the original price of the article.

F92B24 Five pennies, five nickels, and five dimes, are in a box. Three coins are drawn at random without replacement. Find the probability that the total value of these coins is less than 15 cents.

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CONTEST 5

PART 1: 10 MINUTES

F92B25 The length of the tangent to a circle from an external point is 7. If the radius of the circle is 3, find the shorted distance from the point to the circle.

F92B26 If $\log_{10} x = 2 - 2\log_{10} 2$, compute *x*.

PART 2: 10 MINUTES

F92B27 Susan drives to a distant city and returns using the same route. She must average 40 miles per hour both ways in order to keep an appointment in her home city. She is delayed enroute, and can only average 30miles per hour on the trip to the distant city. At what rate must she travel on the return trip to keep the appointment?

F92B28 In a circle with radius 10, two parallel chords are drawn on opposite sides of the center, each 5 units from the center. Find the area between the chords within the circle.

PART 3: 10 MINUTES

F92B29 Express the value of $\sin 285^\circ$ in radical form.

F92B30 Find the largest positive integer p for which $\frac{3p+25}{2p-5}$ is a positive integer.

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CONTEST 1 - SOLUTIONS

F92B1 $\dfrac{\log 5}{\log(1/5)} = \dfrac{\log 5}{\log 1 - \log 5} = \dfrac{\log 5}{-\log 5} = -1$

F92B2 Using the formula for the sum, $S = \dfrac{n}{2}(2r + (n-1)2r)$ which yields $S = rn^2$.

F92B3 $\sqrt{32 - x^2} = -x$. Squaring both sides, we obtain $32 - x^2 = x^2$, which gives $x = \pm 4$. However, 4 does not satisfy the equation. Therefore, -4 is the only root.

F92B4 Let n be the number of teams in Division A.
$_nC_2 = {}_{25-n}C_2 + 36$
$n(n-1)/2 = (25-n)(24-n)/2 + 36$
$48n - 672 = 0$
Thus, $n = 14$.

F92B5 Since OE is perpendicular to AC, it bisects arc AC. Triangle BOE is isosceles, with vertex angle 155^0. The base OBE is therefore equal to 12 ½ ^{0}.

F92B6 Factoring yields $(2^x + 3^y)(2^x - 3^y) = 55$
Since $55 = 11 \cdot 5$ or $55 \cdot 1$, the only two possibilities are

$2^x + 3^y = 11$	$2^x + 3^y = 55$
$2^x - 3^y = 5$	$2^x - 3^y = 1$
$2 \cdot 2^x = 16$	$2 \cdot 2^x = 56$
$x = 3$ and $y = 1$	impossible

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CONTEST 2 - SOLUTIONS

F92B7 The sum of the 9 tests is (9)(92)=828. The sum of the 7 tests is (7)(94)=658. Therefore, the sum of the other 2 tests is 170. If one is 96, the other is 74.

F92B8 Solving for y, y = (500-3x)/5. For y to be an integer, x must be a multiple of 5. Therefore, x can take the value of any multiple of 5, from 5 to 165, or 33 values in all. If x were larger than 165, y would be negative.

F92B9 1/(x+1)=x-1. This yields $x^2 - 1 = 1$ or $x = \pm\sqrt{2}$.

F92B10 If a number is broken into prime factors, $p_1^{k1} \cdot p_2^{k2} \cdot p_3^{k3} \cdot$...., the total number of factors is $(k_1+1)(k_2+1)(k_3+1)...$. Since 1728 = $2^6 \cdot 3^3$, the number of factors is (7)(4) = 28.

F92B11 Factoring, (x- ¼)(x- ¼ + x –1/8) = 0. Thus, x = ¼, 3/16.

F92B12 A zero will be produced by every factor of 12, or $2^2 \cdot 3$. Since 12! Contains $2^{10} \cdot 3^5$, there will be 5 factors of 12, or 5 zeros.

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CONTEST 3 - SOLUTIONS

F92B13 The points of intersection are (0, 7) and (1, 6). The distance is therefore $\sqrt{2}$.

F92B14 Let x= a leg of the isosceles triangle. Then $4 - 2x = x\sqrt{2}$. x($\sqrt{2}$ +2) = 4. Hence, x = $\dfrac{4}{2+\sqrt{2}} = \dfrac{4(2-\sqrt{2})}{2} = 4-2\sqrt{2}$.

F92B15 $\dfrac{1}{r_1}+\dfrac{1}{r_2}=\dfrac{r_1+r_2}{r_1 \cdot r_2}=-\dfrac{p}{q}$.

F92B16 The number is obviously between 2 and 3. Letting x = 2 produces 18. Increasing x gradually, x = 11/5 produces 19, x = 7/3 produces 20, and x = 12/5 is the smallest value that produces 21.

F92B17 $\sin 2x = 2\sin x\cos x$. Since $\cos x = \pm\dfrac{\sqrt{15}}{4}$, depending on whether x is in the first or second quadrant, $\sin 2x = \pm\dfrac{\sqrt{15}}{8}$.

F92B18 Constructing the perpendicular bisectors of the chords, they meet in the center of the circle. $r^2 = 4^2 + \left(\dfrac{1}{2}\right)^2$. Therefore, $r = \dfrac{\sqrt{65}}{2}$.

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CONTEST 4 - SOLUTIONS

F92B19

$7^{3x} = 6^3$
$7^x = 6$
$7^{-x} = 1/6$

F92B20

After two hours, the old machine will have done 2/6 of the job, the new one, 2/5 of its job. 2/6 + 2/5 + x/5 = 1
x = 4/3.

F92B21

The octagon has 8 triangles, each with area $\frac{1}{2}\cdot 10\cdot 10\cdot \sin 45^\circ$.

$8\cdot\frac{1}{2}\cdot 10\cdot 10\cdot\frac{\sqrt{2}}{2} = 200\sqrt{2}$.

F92B22

Using the information given, a + ar = 3 and a/(1-r) = 4. Then a = 4 –4r, and substituting in the first equation, 4 – 4r + (4 – 4r)r = 3, $4r^2 = 1$, $r = \pm\frac{1}{2}$. If $r = \frac{1}{2}$, a = 2, and if $r = -\frac{1}{2}$, a = 6.

F92B23

Letting the original cost be *x*, $x + (.01)Px - .01P(x + .01P) = 1$. Multiplying by 100 twice, $10000x - P^2x = 10000$. $x = \frac{10000}{10000 - P^2}$.

F92B24

The total number of ways that 3 coins can be picked is ${}_{15}C_3 = 455$. The number of ways that the total is less than 15 cents is:
3 pennies ${}_5C_3 = 10$
2 pennies and 1 dime ${}_5C_2 \cdot 5 = 50$
2 pennies and a nickel ${}_5C_2 \cdot 5 = 50$
1 penny and 3 nickels ${}_5C_2 \cdot 5 = 50$
The probability is $\frac{50+50+50+10}{455} = \frac{160}{455} = \frac{32}{91}$.

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CONTEST 5 - SOLUTIONS

F92B25 $x+3=\sqrt{49+9}$
$x=\sqrt{58}-3$

F92B26 $\log_{10} = \log_{10} 100 - \log_{10} 4 = \log_{10}(100/4) = \log_{10} 25$
$x = 25$

F93B27 Let D be the one way distance. T = D/R.
D/40 + D/40 = D/30 + D/x
2Dx = 120D
x = 60.

F92B28 The area is the sum of two 60° sectors and two triangles.
$2\cdot\frac{1}{6}\cdot 100\pi \;+2\cdot 25\sqrt{3} = \frac{100\pi}{3}+50\sqrt{3}$.

F92B29 Using the sum formula, $\sin 285^\circ = \sin(240^\circ + 45^\circ) =$
$= \sin 240^\circ \cos 45^\circ + \cos 240^\circ \sin 45^\circ = -\frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} - \frac{1}{2}\frac{\sqrt{2}}{2} = \frac{-\sqrt{6}-\sqrt{2}}{4}$.

F92B30 If $\frac{3p+25}{2p-5} = N$, then $3p+25=kN$ and $2p-5=k$. Taking these two equations and eliminating p, $k(2N-3)=65$. Since 1, 5, 13 and 65 are the only factors of 65m k can take these values only. Since $p=\frac{k+5}{2}$, the maximum for p is 35.