NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

Sophomore-Freshman Division

CONTEST NUMBER 1

PART I *FALL, 2004* *CONTEST 1* *TIME: 10 MINUTES*

F04SF1 Compute the largest positive integer less than 100 that has exactly 3 positive integral factors.

F04SF2 $a+b+c=2001,\ a+b+d=2002,\ a+c+d=2003,\ b+c+d=2004$
Compute: $d-c+b-a$.

PART II *FALL, 2004* *CONTEST 1* *TIME: 10 MINUTES*

F04SF3 Compute the total number of squares in a 5 row by 5 column checkerboard.

F04SF4 A car tire was punctured by two pieces of glass. The first puncture alone would cause the tire to be entirely flat in 6 minutes. The second puncture alone would cause the tire to be entirely flat in 9 minutes. Assuming the air leaks out at a constant rate, compute the number of minutes it would take for both punctures together to make the tire entirely flat.

PART III *FALL, 2004* *CONTEST 1* *TIME: 10 MINUTES*

F04SF5 Compute: $\dfrac{600^2}{301^2-299^2}$.

F04SF6 $\left(x+\dfrac{1}{x}\right)^2=5$. Compute: $\left|x^3+\dfrac{1}{x^3}\right|$.

ANSWERS:

F04SF1	**49**
F04SF2	**2**
F04SF3	**55**
F04SF4	$\mathbf{\frac{18}{5}}$
F04SF5	**300**
F04SF6	$2\sqrt{5}$

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CONTEST NUMBER 2

PART I *FALL, 2004* *CONTEST 2* *TIME: 10 MINUTES*

F04SF7 Mr. Po tutored his son in algebra. In order to motivate his son, Mr. Po gave him \$8 for each correct equation he solved and fined him \$5 for each incorrect solution. After 26 problems neither owed money to the other. Compute the number of equations his son solved correctly.

F04SF8 A regular polygon of n sides has n diagonals. Compute n.

PART II *FALL, 2004* *CONTEST 2* *TIME: 10 MINUTES*

F04SF9 Compute the largest positive integer less than 325 that has exactly 3 proper factors. (A proper factor is a factor other than the number itself)

F04SF10 In right ΔABC, $\angle C$ is the right angle, the hypotenuse is 6 and the sum of the two legs is 8. Compute the area of ΔABC.

PART III *FALL, 2004* *CONTEST 2* *TIME: 10 MINUTES*

F04SF11 The price of a new car increases by 10%. To the nearest integer, by what percent must the new price be decreased to equal the original price?

F04SF12 Compute all real x such that: $\sqrt{x+1}+\sqrt{x+4}=\sqrt{x+9}$.

ANSWERS:

F04SF7	**10**
F04SF8	**5**
F04SF9	**323**
F04SF10	**7**
F04SF11	**9**
F04SF12	**0**

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CONTEST NUMBER 3

PART I *FALL, 2004* *CONTEST 3* *TIME: 10 MINUTES*

F04SF13 Two natural numbers differ by 2, and their squares differ by 100. Find the smaller of the two numbers.

F04SF14 The Zhang family consists of a father, a mother, and 3 daughters. The sum of all their ages is 90. The daughters were born 2 years apart. The mother's age is 10 more than the sum of the ages of the 3 daughters. The father's age minus the mother's age is the age of the middle daughter. Compute the age of the youngest daughter.

PART II *FALL, 2004* *CONTEST 3* *TIME: 10 MINUTES*

F04SF15 Sonia picked 1/5 of the pears in the orchard. Jesse then picked 1/6 of the pears left by Sonia. There were 150 pears left after Jesse finished. How many pears were in the orchard to begin with?

F04SF16 A 20 pound sponge is 99% water. Some of the water evaporates and the sponge is now 98% water. Compute the weight of the sponge after the evaporation.

PART III *FALL, 2004* *CONTEST 3* *TIME: 10 MINUTES*

F04SF17 Compute x such that: $\dfrac{9!}{3\cdot 6\cdot 9\cdot 12\cdot 15\cdot 18\cdot 21\cdot 24\cdot 27} = x^9$. $(n! = 1\cdot 2\cdot 3\cdot \ldots \cdot n)$

F04SF18 $x + y + z = 20$. Compute the number of ordered triples of positive integers (x, y, z) such that $x < y < z$.

ANSWERS:

F04SF13	**24**
F04SF14	**5**
F04SF15	**225**
F04SF16	**10**
F04SF17	**1/3**
F04SF18	**24**

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Sophomore-Freshman Division

CONTEST NUMBER 1

Fall 2004 Solutions

F04SF1 To have exactly 3 factors, the number must be a perfect square of a prime, since 1 and the number are the other 2 factors. Thus the answer is **49**.

F04SF2 Adding all the left sides and right sides of the equations and dividing by 3 gives us: $a+b+c+d=2670$. Thus, $a=666, b=667, c=668, d=669$. The answer is **2**.

F04SF3 An organized way of solving is to first count the number of squares with side of one box. This is 25. Then for squares with sides of 2, 3, 4, and 5 boxes, we get 16, 9, 4, and 1 respectively. (see a pattern!) The total is **55**.

F04SF4 The rates of deflation through the two punctures are $\frac{1}{6}$ and $\frac{1}{9}$. The combined rate is $\frac{1}{6}+\frac{1}{9}=\frac{5}{18}$. It will take $\mathbf{\frac{18}{5}}$ minutes.

F04SF5 $$=\frac{600\cdot 600}{(301-299)(301+299)}=\frac{600\cdot 600}{2\cdot 600}=\mathbf{300}.$$

F04SF6 $$x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)\left(x^2-1+\frac{1}{x^2}\right) \text{ and } \left(x+\frac{1}{x}\right)^2=x^2+2+\frac{1}{x^2}=5\rightarrow x^2+\frac{1}{x^2}=3.$$

Thus $$\left(x+\frac{1}{x}\right)\left(x^2-1+\frac{1}{x^2}\right)=\pm\sqrt{5}\cdot 2=\pm 2\sqrt{5}\rightarrow\left|\pm 2\sqrt{5}\right|=\mathbf{2\sqrt{5}}$$

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CONTEST NUMBER 2

Fall 2004 Solutions

F04SF7 Let x be the number of correct answers. Then $8x-5(26-x)=0$, $13x=130$ and $x=\mathbf{10}$.

F04SF8 The number of diagonals of an n-gon is $\frac{n(n-3)}{2}$. Thus,

$$\frac{n(n-3)}{2}=n \rightarrow n^2-3n=2n \rightarrow n=\mathbf{5}.$$

F04SF9 To have exactly 3 proper factors, the number must be the product of two primes or must be the cube of a prime number. It must also be as large as possible. The positive square root of 325 is less than 19, so trying primes on both sides, we try 18^2-1, yielding 19 x 17 = **323**. (5^3 is too small and 7^3 is too large)

F04SF10 $a+b=8$, so $64=(a+b)^2=a^2+b^2+2ab=c^2+2ab=36+2ab$, so $2ab=28$. The area of the triangle is $\frac{1}{2}ab=\frac{2ab}{4}=7$.

F04SF11 Let the price of the car be p and the percent we are looking for be x. Now, $(p+.1p)-.01x(p+.1p)=p$. Solving and dividing by p, we get,

$$1.1-.01x(1.1)=1 \rightarrow x=\frac{100}{11}\approx \mathbf{9}.$$

F04SF12 Squaring both sides, we get:

$$x+1+x+4+2\sqrt{(x+1)(x+4)}=x+9 \rightarrow$$

$$x-4=-2\sqrt{(x+1)(x+4)} \rightarrow$$

$$x^2-8x+16=4(x^2+5x+4) \rightarrow$$

$$x=0,-\frac{28}{3}$$

Trying $-\frac{28}{3}$, we reject this as an extraneous root, and the answer is **0**.

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CONTEST NUMBER 3

Fall 2004 Solutions

F04SF13 Let x and x + 2 be the two numbers. $(x+2)^2 - x^2 = 100$. Solving gives x = **24**.

F04SF14 Let f, m, d_1, d_2, d_3 = the ages of the father, mother and the three daughters

$$f + m + d_1 + d_2 + d_3 = 90,\ d_1 = d_2 - 2,\ d_3 = d_2 + 2$$

$$m = 10 + d_1 + d_2 + d_3 = 3d_2 + 10 \rightarrow f + 6d_2 + 10 = 90 \rightarrow f = 80 - 6d_2$$

$$f - m = d_2 \rightarrow f = m + d_2 = 4d_2 + 10$$

$$80 - 6d_2 = 4d_2 + 10 \rightarrow d_2 = 7 \rightarrow d_1 = \mathbf{5}$$

F04SF15 The 150 pears left represents 5/6 of the number Jesse found there, so Jesse found (6/5)(150) = 180 pears. This represents 4/5 of the number Sonia found originally, so there was at first (5/4)(180) = **225** pears.

F04SF16 The sponge originally contains 20 x 1% = .2 pounds of material. If x is the weight of the sponge after evaporation, $.2 = .02x \rightarrow x = \mathbf{10}$.

F04SF17 The fraction reduces and we get: $\frac{1}{3^9} = x^9 \rightarrow x = \mathbf{\frac{1}{3}}$.

F04SF18

$x=1$	$y+z=19$	$2 \le y \le 9,\ 10 \le z \le 17$	8 triples
$x=2$	$y+z=18$	$3 \le y \le 8,\ 10 \le z \le 15$	6 triples
$x=3$	$y+z=17$	$4 \le y \le 8,\ 9 \le z \le 13$	5 triples
$x=4$	$y+z=16$	$5 \le y \le 7,\ 9 \le z \le 11$	3 triples
$x=5$	$y+z=15$	$6 \le y \le 7,\ 8 \le z \le 9$	2 triples

We thus have **24** triples.